

Flux dependence of mixing depth during morning fumigation or unstable condition

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Abstract : Doppler sodar derived inversion depth and mixing depth pertaining to monsoon 'onset' phase of year 1990 are normalised by surface layer Monin-Obukhov length. A log-log best fit of normalised data suggests a coefficient to be a reducing factor on inversion height to yield a mixing height in ABL. The coefficient is able to be expressed as a non-dimensional form of few energy fluxes in ABL. The coefficient also has a scatter around experimental best-fit that can be explained with a surface stability parameter dependence.

Keywords : Monsoon, micrometeorology, sodar, fumigation, convection.

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1. Introduction

During the monsoon months of 1990, an Aerovironment make Doppler sodar (Model – 2000) was operated in the campus of Indian Institute of Technology, Kharagpur, West Bengal (22.3° N, 87.2°E) as part of MONTBLEX*-1990 [1]. A coordinated multi-institutional network of experimental stations covering almost entire northern part of India was achieved. Besides the 3-axis monostatic Doppler sodar (Sound Detection and Ranging System) at Kharagpur, there was a 30-metre high 6-level micrometeorological mast, each level was instrumented with various fast as well as slow response sensors located on booms fixed at 1, 2, 4, 8, 15 and 30 metres above ground. Sensors were facing southward. The site was mostly an uncultivated fetch of land with short and tall grass. The terrain slope of experimental site was about 10^{-2} over 400 Km². Mean wind direction in the region was $190 \pm 15^\circ$. Background noise while sounding was mostly due to railway traffic; low flying small aircrafts, birds and insects.

*Acronym stands for Monsoon Trough Boundary Layer Experimental that was sponsored by the Department of Science & Technology, Government of India.

The said sodar provided continuous one hour average values of inversion depth (Z_i) and Atmospheric Boundary Layer (ABL) mixing depth (l_m). In the present work, only 14 days' data pertaining to the period between June 3 and 24, 1990 are analysed. Data utilised pertains to time periods between 0500 and 1700 hours. The purpose of present work is to present a functional relationship between l_m and Z_i in non-dimensional form.

It is found from available sodar data that when l_m and Z_i are non-dimensionalised with the help of Monin-Obukhov length (L_0) representing shear and buoyancy measures in the surface layer of ABL, there exists a relationship of the form

$$\frac{l_m}{L_0} = A_{1,2} \left(\frac{Z_i}{L_0} \right)^{C_{1,2}}, \quad (1)$$

where subscripts 1 and 2 of A and C represent stable and unstable regimes of ABL respectively. A_1 , A_2 , C_1 and C_2 are all found to be constants.

L_0 is evaluated from slow-response data of mast, following a procedure discussed in Section 2 and after averaging the basic input data further for 60 minutes. It is given by the relation

$$L_0 = \frac{u_*^2}{k(g/\theta_{v0})\theta_{v*}}, \quad (2)$$

where θ_{v0} is the virtual potential temperature at the surface of earth and various other parameters in L_0 are in standard notation.

It is found that A_1 and C_1 have different sets of values for stable situations under morning fumigation conditions and under unstable conditions. The values of $A_{1,2}$ and $C_{1,2}$ are obtainable from best-fit curves of entire data-set.

It is however, found from Pearson's correlation study that l_m correlates with various fluxes of heat energy. An attempt has been made to express $A_{1,2}$ as non-dimensional form of various fluxes. In fact, various forms of $A_{1,2}$ for stable conditions in morning fumigation periods as well as for convectively unstable situations may be found out. Here, one form of A_1 for stable and another form of A_2 for unstable conditions are presented. The sodar measured value of l_m is compared with calculated value in terms of fluxes and non-dimensional inversion height. It is also to be noted that whenever the flux terms involve Z_i , the value is taken for the present sodar supplied data.

Lastly, the values of A_1 and A_2 are computed using respective flux forms and the ratio of them with corresponding experimentally obtained average values of A_1 and A_2 denoted by $[A_1]$ and $[A_2]$, is calculated. From a plot between the said ratio (ordinate) and surface stability (z/L_0), it is found that the ratio has a scatter around 1 and the scatter can be expressed as separate polynomial of (z/L_0) in the two conditions. (z/L_0) represents surface layer stability parameter where z is surface layer depth and is taken as 10 % of Z_i .

2. Surface parameters from 30 m mast data

The surface parameters like θ_{v*} , u_* and heat flux in kinematic form are related as

$$H_1 = -u_* \theta_{v*}, \quad (3)$$

u_* and θ_{v*} are determined using a linear super-imposition method for wind velocity and temperature profile in surface layer of ABL [2] and choosing standard similarity relations [3] for both, stable and unstable situations.

Once θ_{v*} and u_* are evaluated, L_0 may then be calculated using expression (2).

3. Relation between non-dimensional l_m and non-dimensional Z_i from sodar data

$\log_e (l_m / |L_0|)$ is plotted against $\log_e (Z_i / |L_0|)$ for stable situations under morning fumigation conditions and for the unstable situations. It is found that the best-fit straight line under stable condition (Figure 1(a)) is given as,

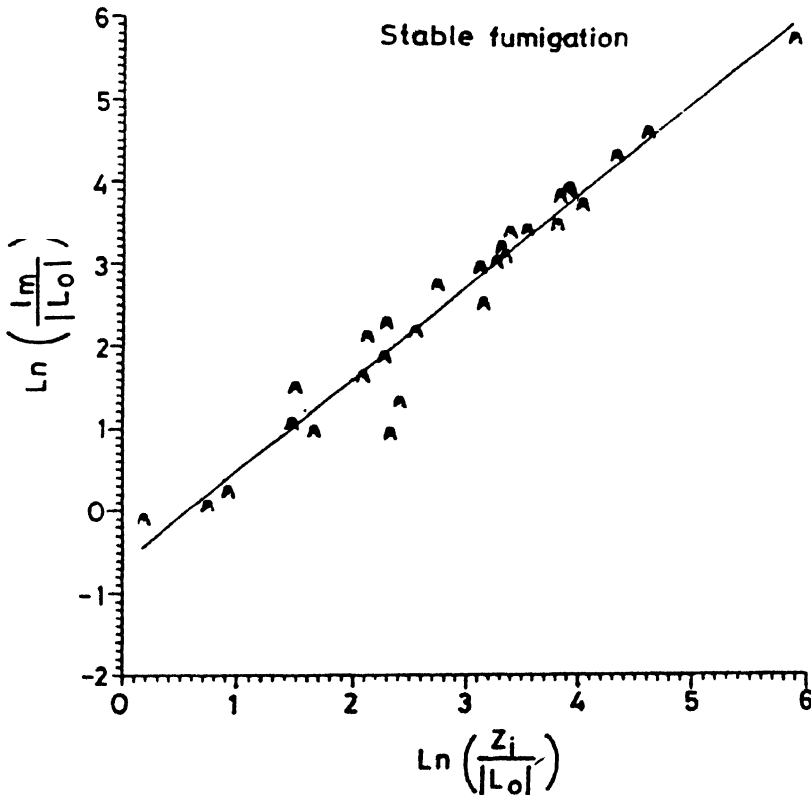


Figure 1. (a) Data points in the fit as defined in eq. (4) for the stable fumigation conditions of ABL. L_0 is obtained every 60 minutes and then the depths are non-dimensionalised.

$$\log_e \left(\frac{l_m}{|L_0|} \right) = 1.12001 \log_e \left(\frac{Z_i}{|L_0|} \right) - 0.627951$$

or

$$\frac{l_m}{|L_0|} = 0.533 \left(\frac{Z_i}{|L_0|} \right)^{1.120} \quad (4)$$

In this case, the data points have some scatter about the best fit straight line. So, under stable fumigation conditions $[A_1] = 0.533$.

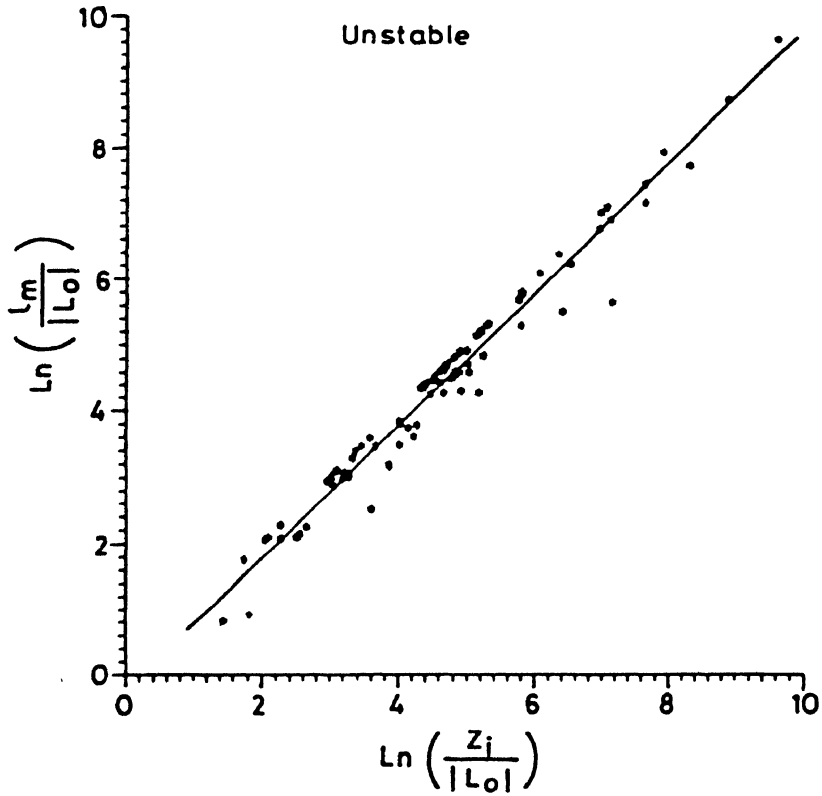


Figure 1. (b) Same as in Figure 1(a) but for the unstable ABL cases (eq. 5); notice closeness of points.

In unstable condition, corresponding best-fit curve is given by (Figure 1b),

$$\log_e \left(\frac{l_m}{|L_0|} \right) = 0.99729 \log_e \left(\frac{Z_i}{|L_0|} \right) - 0.212$$

or,

$$\frac{l_m}{|L_0|} = 0.809 \left(\frac{Z_i}{|L_0|} \right)^{0.99729} \quad (5)$$

As evident from Figure 1(b), the scatter of data points is much less in this case and $[A_2] = 0.809$.

4. Pearson's correlation study of l_m and various flux forms of heat

The net radiation R is available from a Funk-type net radiometer connected to one of the analog input channels of Doppler sodar electronics and is archived in one hour average form in an electrical unit. For expressing net radiational heat flux in kinematic form, it is taken as $r_0 R$. Here r_0 is taken as a conversion constant and is assumed that the electrically averaged net radiation is linear with the radiational heat flux in kinematic form.

Let us introduce another form of heat flux in kinematic form

$$H_2 = \frac{u_*^3 \theta_{v*}^{ml}}{W_*^2}, \quad (6)$$

where θ_{v*}^{ml} is the scaling virtual potential temperature occurring in the entire mixed layer of ABL and $W_* = (g/\theta_{v0} u_* \theta_{v*} Z_i)^{1/3}$ is the convective velocity scale [4]. W_* may be evaluated using the above expression and then θ_{v*}^{ml} may be calculated using relation $\theta_{v*}^{ml} = H_1/W_*$.

$$\text{Similarly, } H_3 = \frac{u_*^2 \theta_{v*}^{ml}}{W_*} \quad (7)$$

represents another form of kinematic heat flux. One more form of kinematic heat flux was introduced [5] as

$$H_4 = \frac{\sigma_w^3}{\alpha^{3/2} Z_i \beta}, \quad (8)$$

σ_w is the standard deviation of vertical wind speed and this magnitude is directly available from sodar. $\alpha^{3/2}$ is a constant and its value is found to be 1.76 ± 0.713 for unstable situations.

Another heat flux may also be introduced in the form of H_4 as

$$H_5 = \frac{(\sigma_w^3)_{Z_i/2}}{\alpha^{3/2} Z_i/2 \beta}, \quad (9)$$

at around half ABL depth with $(\sigma_w^3)_{Z_i/2}$ here representing average of σ_w values recorded at depths ranging between 40% and 60% of Z_i . g is acceleration due to gravity and $\beta = g/\theta_{v0}$ as explained earlier. Obviously, H_4 and H_5 have significance only under unstable conditions.

In fact, correlation studies were performed between l_m and various positive powers of the above mentioned six forms of heat fluxes, namely, R , H_1 , H_2 , H_3 , H_4 and H_5 and it is found that the correlation coefficient varies from very strong to an appreciable magnitude for various positive powers of the fluxes. There exists both positive as well as negative correlation coefficients.

In morning fumigation hours of the stable ABL case, correlation studies of l_m were undertaken with various powers of R , H_1 , H_2 and H_3 , whereas for unstable cases correlation

studies were done with various powers of R , H_1 , H_4 and H_5 . Since l_m is correlated with various forms of heat fluxes, it is assumed that A_1 or A_2 should be expressible in terms of various powers of heat flux, such that the entire expression becomes a non-dimensional quantity. Obviously, to make the entire expression non-dimensional, l_m must have both positive and negative correlations.

Then, the heat flux functional form of A_1 can be written as

$$\exp(K_1) = D_1 \frac{(r_0 R)^4 (H_1)^{1.5}}{(H_2)^{2.5} (H_3)^{3.0}},$$

D_1 is an adjustable constant to make $\exp(K_1)$ numerically equal to A_1 . The power of fluxes is such that the correlation with l_m is maximum. In the expression of $\exp(K_1)$, the fluxes coming in the denominator have a negative correlation.

D_1 may be absorbed within $r_0 R$ and then r_0 may be evaluated using the following procedure—one can start with an initial value of r_1 , say 0.001 and then go on increasing the value of r_1 in a step of 10^{-3} till $\exp(K_1)$ converges within an accuracy of 10^{-4} . The value of r_0 for which $\exp(K_1)$ converges, is taken as the necessary value.

Then, it is found using the experimental data set that

$$\exp(K_1) = \frac{(3.274R)^{4.0} (H_1)^{1.5}}{(H_2)^{2.5} (H_3)^{3.0}}. \quad (10)$$

One can now write the proposed expression for l_m in terms of Z_i and flux forms in stable fumigation conditions as

$$\frac{l_m}{|L_0|} = \frac{114.9(R)^{4.0} (H_1)^{1.5}}{(H_2)^{2.5} (H_3)^{3.0}} \left(\frac{Z_i}{|L_0|} \right)^{1.12}. \quad (11)$$

An identical procedure may be followed in finding the form of A_2 in unstable conditions. It is found that the conversion factor for R during unstable conditions is 0.486 (inclusive of factor D_1) and then using maximum correlated powers of heat fluxes in unstable cases it can be written as

$$\exp(K_2) = \frac{(0.486R)^{6.0} (H_4)^{1.6}}{(H_1)^{7.4} (H_5)^{0.20}}. \quad (12)$$

The proposed expression for l_m in terms of Z_i and flux forms in unstable ABL may then be written as

$$\frac{l_m}{|L_0|} = \frac{0.0558(R)^{6.0} (H_4)^{1.6}}{(H_1)^{7.4} (H_5)^{0.2}} \left(\frac{Z_i}{|L_0|} \right)^{0.997}. \quad (13)$$

5. Results and conclusions

The Doppler sodar obtained values of l_m against the derived values of l_m using flux relations are presented in Figures 2(a) and 2(b), for stable fumigation conditions and for unstable

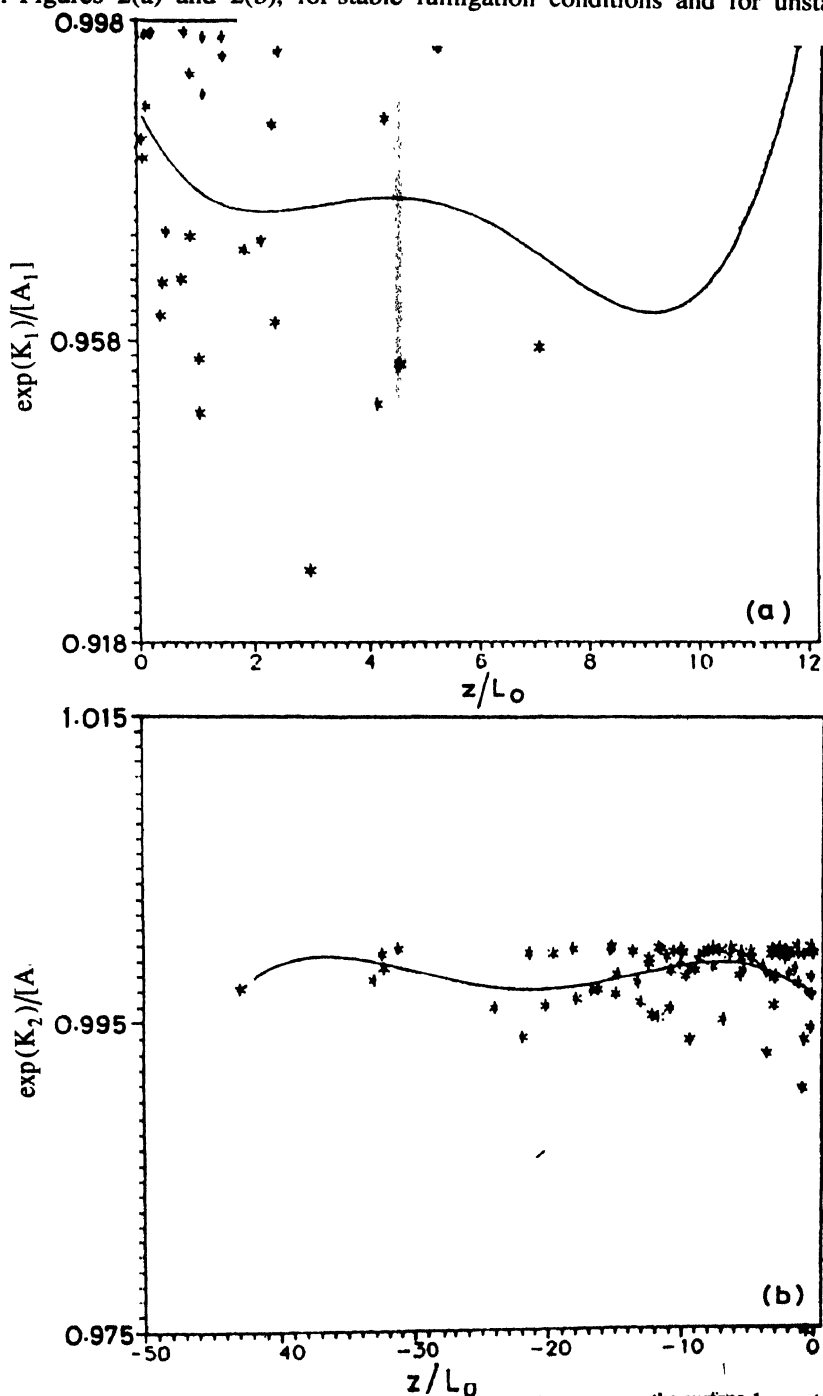


Figure 2. (a) Fifth order polynomial fit of ratio $\exp(K_1)/[A_1]$ versus the surface layer stability parameter for stable fumigation ABL cases in morning hours. (b) Fourth order polynomial fit of ratio $\exp(K_2)/[A_2]$ versus stability parameter for unstable ABL cases.

conditions respectively. It is obvious that the fit is quite good, though there exist some scatter around an ideal straight line form.

Ratio $\exp(K_1)/[A_1]$ is found to have a mean value of 0.98 with a standard deviation of 0.02, whereas $\exp(K_2)/[A_2]$ is found to have a mean value of 0.996 with a standard deviation of 0.005.

When ratio of computed magnitudes of $\exp(K_1)/[A_1]$ and $\exp(K_2)/[A_2]$ are plotted along with the surface layer stability parameter (z/L_0) in the corresponding time intervals, then it is found that there is an oscillatory dependence of both the ratios with the parameter viz. :

For stable fumigation conditions

$$\begin{aligned} \exp(K_1)/[A_1] = & 0.98958 - 0.0167(z/L_0) + 0.0065(z/L_0)^2 \\ & - 0.001(z/L_0)^3 + 4 \times 10^{-3} (z/L_0)^4, \end{aligned} \quad (14)$$

and for unstable conditions

$$\begin{aligned} \exp(K_2)/[A_2] = & 0.9963 - 0.001(z/L_0) - 9.7 \times 10^{-5} (z/L_0)^2 \\ & - 2.6 \times 10^{-6} (z/L_0)^3. \end{aligned} \quad (15)$$

As can be seen in Figures 2(a) and 2(b), oscillatory dependence is more pronounced in the unstable case than in stable fumigation case.

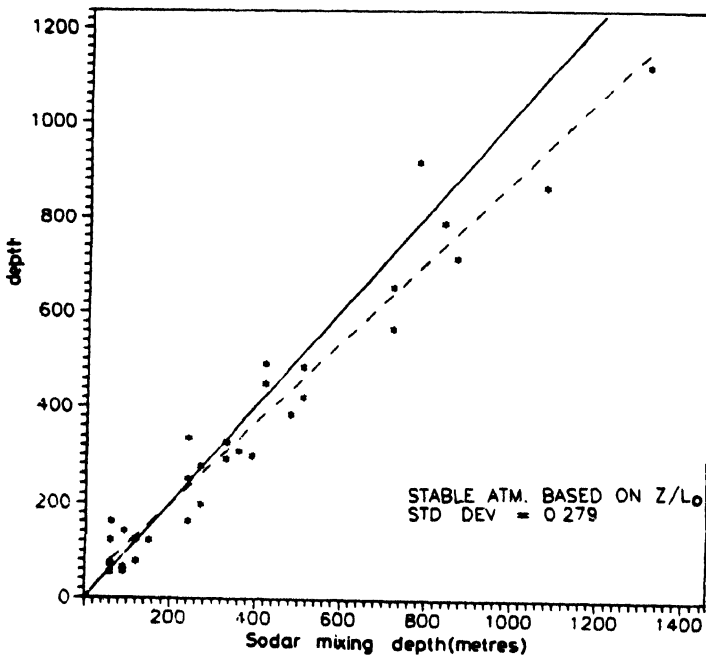


Figure 3. (a) Scatter diagram of l_m that is directly obtained from sodar data sets and that computed using eq. (11) for stable fumigation ABL cases.

This indicates that when applying $\exp(K_1)$ and $\exp(K_2)$ for any reduction of a known Z_i then there really exists a scatter around experimentally obtained value which we consider,

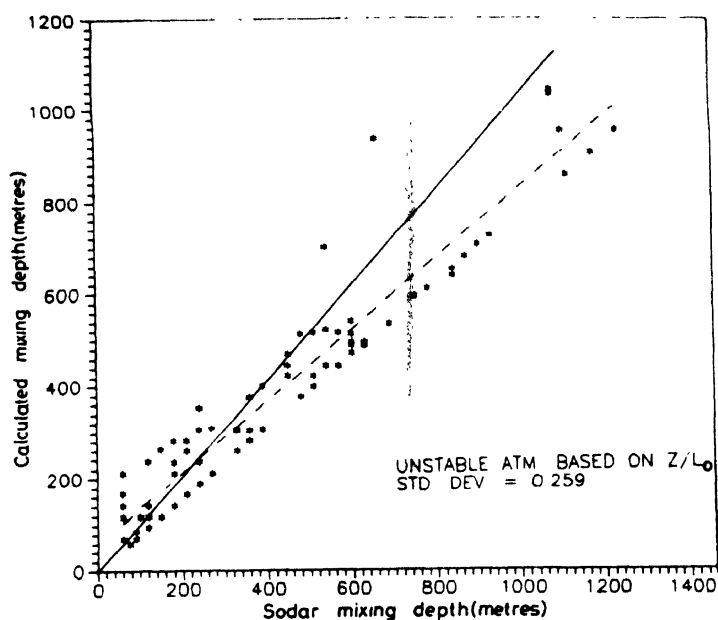


Figure 3. (b) Same as in Figure 3(a) but for unstable ABL cases and using eq (13); A reference line (dashed with slope 1) shown allows us to justify the scatter in data.

is an error in the exponentials around experimental $[A_1]$ and $[A_2]$. However, correction on the exponentials may be done by utilising the polynomial form of (z/L_0) as appearing in the right hand sides of eqs. (14) and (15) respectively.

Acknowledgments

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